

# Optimal Multi-User Scheduling for the Unbalanced Full-Duplex Buffer-Aided Relay Systems

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**Abstract**—Multi-User scheduling is challenging due to the channel unbalance problem, which leads to system performance degradation. In this paper, two optimal multi-user scheduling schemes maximizing the system throughput are proposed for the fixed and adaptive power transmission scenarios of the full-duplex (FD) multi-user buffer-aided relay system, respectively. Independent and non-identically distributed (i.n.i.d.) model is used to characterize the unbalanced channels of different links. In particular, the optimal weight factor of each pair is designed based on the statistical channel state information in both scenarios. With the weight factors, the proposed schemes are able to balance the throughput gaps between different links. In addition, we propose an optimal power allocation scheme with closed-form expressions under average power constraint for the adaptive power transmission scenario. By combining the optimal weight factor and the power allocation scheme, novel optimal selection function is obtained to facilitate the selection process. Considering the specific i.n.i.d. Rayleigh fading, the system throughput is further derived in both cases. Theoretical analysis is verified by the numerical simulations and the results demonstrate the superiority of the proposed schemes.

**Index Terms**—Buffer-aided relay, full-duplex mode, multi-user scheduling, power allocation, unbalanced channels.

## I. INTRODUCTION

THE development of wireless communication technologies, including the Internet of Things, vehicular-to-everything communication and so on, predict one of the key attributes, i.e., dense user environment, of the future wireless communication systems. In the fifth generation wireless communication systems, it is envisioned the number of the connected devices will increase 10-100 times over the

current paradigms [2]. Under dense user environment, user scheduling schemes play important roles in improving the system performance, especially when the system resource is limited. When multiple users need to contend for the same radio resources, selecting the best user is critical to improve the system performance, e.g., achievable rate, outage probability or latency. On the other hand, the relaying technology is expected to extend coverage of cellular systems and improve the throughput of cell-edge users [3]. In addition, it is a cost-effective solution by deploying relay nodes in the hot-areas, i.e., dense user areas, to unload the traffic pressure of the base stations [4]. In this context, it is urgent to design an effective multi-user scheduling scheme to allocate the limited resource to multiple users. However, this is a challenging problem due to the unbalanced channel qualities, which leads to different throughput of different links. This paper is dedicated to proposing two optimal multi-user scheduling schemes for the full-duplex (FD) buffer-aided multi-user relay system under the fixed and adaptive power transmission scenarios, respectively. Numerical simulations validate the effectiveness of the proposed schemes.

### A. Background and Motivation

A lot of efforts have been made on the topic of multi-user relay systems, where multiple sources intend to communicate with pre-paired multiple destinations via the relay node [5]–[11]. The practical scenario of the considered system includes the relay-assisted device-to-device communication, where multiple devices exchange information with another set of devices under the help of the relay node [12]. In [6], the sum rate of the multiple-input multiple-output (MIMO) relay system with multiple source-destination pairs was analyzed. The power allocation algorithms were studied in [7] to 1) minimize the total transmit power of the sources and the relay under the required signal-to-interference-plus-noise ratio (SINR) constraint of each pair; 2) maximize the minimum SINR of all source-destination pairs subject to the given total power budget. In [8], the authors proposed a multi-user scheduling scheme, where multiple users were selected simultaneously, to minimize the multi-user interference and achieve the diversity gain. Later, the massive MIMO approach, where the number of antennas at the relay node was assumed to approach to infinity, was investigated for the multi-user relay system. [9], [10] studied the spectral efficiency and energy efficiency of the massive MIMO multi-user relay system. The users' ergodic rates of the massive MIMO multi-user relay system were analyzed in [11]. In addition to the

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MIMO relay systems, in [13], [14], the authors studied the achievable rate and outage probability of the single antenna multi-user relay system. The Max link selection scheme was considered and the results verified that the multi-user diversity gain could be obtained to improve the system performance. The norm based and minimal distance based schemes were proposed in [15] to select users to exchange information.

In recent years, the co-time co-frequency FD mode, which enables the transmitting and receiving simultaneously within the same frequency band, has been proposed [16]–[18]. The experimental results showed that the self-interference incurred by the FD mode can be suppressed to the noise level [18]–[20]. The authors in [21] showed that the FD mode has the potential to double the rate of the HD counterpart. For the FD relay system, [22], [23] proposed the Max-Min multi-user scheduling scheme under the independent but non-identically distributed (i.n.i.d.) fading environment. The results demonstrated that the system performance is degraded by the channel unbalance problem since the poorer links serve as the bottleneck of improving the system performance. To achieve more flexible link selection, some researchers considered buffers at the relay node. In [24], the authors studied the buffer-aided relay system and proposed an adaptive link selection scheme. In [25], [26], the Max-Max link selection scheme and Max link selection scheme based on the instantaneous SINR were proposed. However, even with the consideration of buffers, these link selection schemes in [24]–[26], which were based on the instantaneous channel state information (CSI), cannot solve the channel unbalance problem. The reason is that due to the randomness, the instantaneous channel coefficients cannot reflect the long-term link qualities. Hence, system performance is restricted by the links with poorer channel qualities (more details are discussed in the section III-A). To the best of the authors' knowledge, how to effectively solve the multi-user scheduling problem under unbalanced fading environment for the multi-user relay system still remains to be solved.

### B. Contributions

This paper is dedicated to solving the channel unbalance problem for the multi-user relay system, aiming at achieving maximum system throughput. In particular, we consider a multi-user FD buffer-aided relay system with i.n.i.d. channel environment, where multiple pairs of users compete for the relay node to exchange information. Two optimal multi-user scheduling schemes are proposed for the considered system under the fixed and adaptive power transmission scenarios, respectively. In the proposed schemes, weight factor of each link is calculated according to the long-term statistical CSI. By designing the optimal weight factors, the proposed scheme can effectively solve the channel unbalance problem and maximize the system throughput. It is noted that the notion of weight factor has been widely used in the proportional fair scheduling (PFS) scheme [27], [28]. However, the PFS scheme is mainly designed for the one-hop link systems such that it cannot fit the two-hop relay systems due to the coupled relationship between the first hop, i.e., source-to-relay, and second hop, i.e., relay-to-destination. Hence, the existence of the PFS scheme does not diminish the contribution of this

paper. In summary, the main contributions of this paper are listed as follows.

- In the fixed power transmission scenario, different from the previous user scheduling schemes which were mainly based on the instantaneous CSI [22], [29], we propose an optimal multi-user scheduling scheme considering both the instantaneous and statistical CSI. In particular, we design the optimal weight factors, which are based on long-term statistical CSI and under the link balance criteria, to assist the user scheduling process. By designing the weight factors, it is able to balance the utility ratios of different links as we can allocate more time slots to the links with poor qualities and less time slots to the links with strong qualities, which is counter-intuitive. In this way, the throughput gaps between the poor links and strong links can be mitigated, bringing in improvement of the overall system throughput.
- In the adaptive power transmission scenario, we consider the system average power constraint, under which the transmit power of the selected user is adaptively designed and the exact closed-form power allocation expressions are obtained. In addition, a joint optimal multi-user scheduling scheme with the power allocation scheme is proposed. Combining the optimal weight factors and the optimal power allocation, novel selection functions, based on which to schedule the users, are obtained. The proposed scheme not only balances different links but also further improves the system throughput compared to that of the fixed power transmission scenario.
- To validate the effectiveness of the proposed schemes, we conduct extensive numerical simulations. First, the weight factors under different cases are demonstrated, which are consistent with the theoretical derivations. In addition, the system throughput of the proposed schemes are compared with that of the previous schemes, including the Max-Min scheme and PFS scheme. The results validate that the proposed schemes can effectively balance the channel disparities between different links. Furthermore, the results show that in the adaptive power transmission scenario, the throughput can be further improved compared to that of the fixed power transmission counterparts.

### C. Outline of the Paper

The rest of this paper is organized as follows. Section II describes the considered system model, CSI requirements and basic transmission scheme. Two optimal scheduling schemes for the fixed and adaptive power transmission scenarios are proposed in section III and section IV, respectively. Section V analyzes the system throughput under Rayleigh fading environment. Numerical simulations are presented in section VI. Finally, section VII concludes this paper.

## II. SYSTEM MODEL

In this section, we elaborate on the physical-layer channel model, CSI requirements and basic transmission scheme.

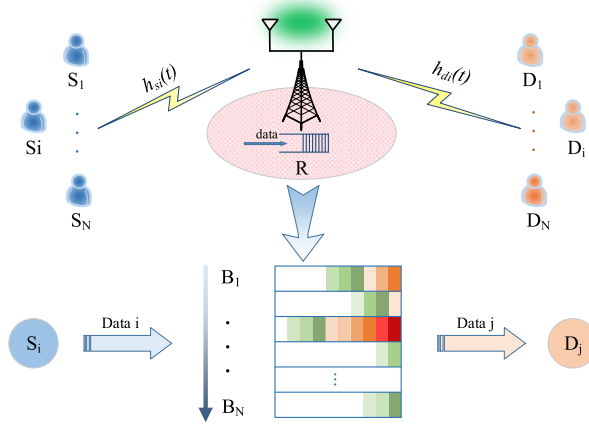


Fig. 1. The multi-user buffer-aided relay system model, which comprises  $N$  pairs of users and a relay node. The source  $S_i$  is sending information to the destination  $D_i$  with the help of the relay node, which is equipped with  $N$  buffers from  $B_1$  to  $B_N$ . The messages from  $S_i$  are first stored in the buffer  $B_i$  and forwarded to  $D_i$  in the later time slots.

### A. Channel Model

As shown in Fig. 1, we consider a single antenna multi-user buffer-aided FD relay system,<sup>1</sup> where all the users are equipped with only one antenna and the relay node is equipped with two antennas, one for transmission and the other for reception, to achieve the FD transmission [20]. The  $N$  pairs of users are pre-defined, i.e.,  $S_i$  sends messages to  $D_i$ ,  $i \in \{1, 2, \dots, N\}$ . It is assumed that the direct links between source nodes and destination nodes do not exist due to the propagation loss, shadowing effects and penetration losses [5]–[7]. The pair-wise communication procedures between  $S_i$  and  $D_i$  can only be established via the relay node, which is equipped with  $N$  buffers<sup>2</sup> from  $B_1$  to  $B_N$ . In this paper, we assume that by using the methods in [18]–[20], the self-interference can be effectively suppressed. Hence, only the residual self-interference (RSI) after suppression is considered in the following analysis. The messages transmitted by the source node  $S_i$  are first stored in the buffer  $B_i$ , and then re-encoded and forwarded to the destination  $D_i$  when the R- $D_i$  link is selected.

We denote the channel coefficients of the links  $S_i$ -R and R- $D_i$  in the  $t$ -th time slot as  $h_{s_i}(t)$  and  $h_{d_i}(t)$ , respectively. We assume that  $h_{s_i}(t)$  and  $h_{d_i}(t)$  are assumed to be i.n.i.d. complex Gaussian random variables. In addition, the duration of each time slot is assumed to be equal to the channel coherent time. Thus, the channels can be viewed as block fading channels. Furthermore, we consider a relatively steady system rather than a highly dynamic network such that the mobility of the mobile users can be neglected. Hence, we assume that the relay node can obtain the statistical CSI, such as mean value, variance and distribution functions, which are used to facilitate the proposed scheduling schemes. The

<sup>1</sup>The utilization of the buffer at the relay node aims to improve the system throughput. The considered system model targeting on the applications without strict delay requirements, for example the multimedia content sharing between user nodes via the relay node during the network off-peak times.

<sup>2</sup>It can be achieved by dividing one buffer into  $N$  logical partitions according to the number of user pairs.

transmit power of the source node  $S_i$  is denoted by  $P_{s_i}(t)$ , the transmit power of the relay node forwarding messages to destination  $D_i$  is denoted by  $P_{d_i}(t)$ , respectively. The instantaneous SINRs of the  $S_i$ -R and R- $D_i$  links in the  $t$ -th time slot are given by  $\gamma_{s_i}(t) \triangleq \varphi_{s_i}(t)g_{s_i}(t)$  and  $\gamma_{d_i}(t) \triangleq \varphi_{d_i}(t)g_{d_i}(t)$ , where  $g_{s_i}(t) = |h_{s_i}(t)|^2$  and  $g_{d_i}(t) = |h_{d_i}(t)|^2$  denote the channel gains of the  $S_i$ -R and R- $D_i$  links, respectively.  $\varphi_{s_i}(t) = \frac{P_{s_i}(t)}{\sigma_s^2 + \sigma_r^2}$  and  $\varphi_{d_i}(t) = \frac{P_{d_i}(t)}{\sigma_{d_i}^2}$  denote the transmit SINRs of the source node  $S_i$  and the relay R, respectively.  $\sigma_r^2$  and  $\sigma_{d_i}^2$  denote the variances of the additive white Gaussian noise at the relay node R and the destination node  $D_i$ , respectively.  $\sigma_s^2$  denotes the variance of the RSI after self-interference cancellation [17], [18]. According to [16]–[20], [30], the effects of the RSI is model as additive Gaussian noise that reduces the received SINR performance. In addition, we denote the average channel gains as  $\Omega_{s_i} \triangleq \mathbb{E}\{g_{s_i}(t)\}$  and  $\Omega_{d_i} \triangleq \mathbb{E}\{g_{d_i}(t)\}$ , and the average SINR as  $\Theta_{s_i} \triangleq \mathbb{E}\{\gamma_{s_i}(t)\}$  and  $\Theta_{d_i} \triangleq \mathbb{E}\{\gamma_{d_i}(t)\}$ . Here, we present the channel capacity expressions of the  $S_i$ -R and R- $D_i$  links as

$$C_{s_i}(t) = \log_2 [1 + \gamma_{s_i}(t)], \quad (1)$$

$$C_{d_i}(t) = \log_2 [1 + \gamma_{d_i}(t)], \quad (2)$$

respectively.

### B. CSI Requirements

To perform the optimal multi-user scheduling scheme, the knowledge of both the instantaneous CSI, i.e., channel quality indicator [31], and statistical CSI, i.e., channel distribution information [32], [33], are needed at the relay node. Similar to the current system design, the instantaneous CSI is acquired at the beginning of each time slot and used for the whole time slot. Each time slot is divided into two phases, CSI acquisition phase and data transmission phase. In the CSI acquisition phase, all the source nodes and destination nodes transmit pilot signals along with the system information, e.g., connection requests, to the relay node in the orthogonal manner. Based on the received signals, the relay node can obtain the instantaneous CSI [34]. It is noted that in order to perform coherent demodulation, pilot signals are needed at the receiver [35]. Hence, there will be no additional overhead on the radio links for CSI estimation. Based on the estimated CSI, the relay node selects the source node and destination node according to the proposed schemes. Then, the data transmission phase begins. In addition, we assume that over a much longer period of time, the relay node can obtain statistical CSI, which provides the distribution knowledge of the random channel variation. Based on the CSI, the relay node can select the best source node and destination node in each time slot according to the proposed schemes. It should be noted that in this paper, we focus on the design of the multi-user scheduling schemes. Thus, the channel estimation error, the CSI outdated problem and the quantization error are omitted.

### C. Basic Transmission Scheme

In this paper, we propose two optimal user scheduling schemes for the multi-user buffer-aided FD relay system under

fixed and adaptive power transmission scenarios. With the FD mode, the relay node can schedule one source to relay link and one relay-to-destination link, simultaneously, according to the selection criteria (refer to Section III and Section IV). Next, we briefly explain the basic transmission schemes used in this paper.

If the source node  $S_i$  is selected to send messages in the  $t$ -th time slot, it transmits with the rate given by [24], [36]

$$R_{s_i}(t) = \min\left\{C_{s_i}(t), \frac{L - Q_i(t-1)}{T_d}\right\}, \quad (3)$$

where  $T_d$  denotes the duration of each time slot.  $L$  is the size of the each buffer. Here, we consider that the source nodes always have data to transmit following [24], [36].

The messages will be first stored in the buffer  $B_i$ . If the destination node  $D_i$  is selected to receive messages in the  $t$ -th time slot, the relay node transmits with the rate given by

$$R_{d_i}(t) = \min\left\{\frac{Q_i(t-1)}{T_d}, C_{d_i}(t)\right\}, \quad (4)$$

We use  $Q_i(t)$  to denote the buffer state of the buffer  $B_i$  in the  $t$ -th time slot. The buffer state is updated by

$$Q_i(t) = Q_i(t-1) + R_{s_i}(t)T_d - R_{d_i}(t)T_d. \quad (5)$$

### III. OPTIMAL USER SCHEDULING SCHEME WITH FIXED POWER TRANSMISSION

In this section, we first review two existing multi-user scheduling schemes for the two-hop relay systems. Then the throughput maximization problem is formulated and the optimal multi-user scheduling scheme is proposed for the considered system under the fixed power transmission scenario.

#### A. Existing User Scheduling Schemes

In the traditional multi-user relay systems with or without buffers, there are mainly two kinds of scheduling schemes. In the next, we briefly review two typical multi-user scheduling schemes: Max-Min user scheduling scheme [22] and Max-Max user scheduling scheme [29], which are both applicable for the considered system in this paper. For simplicity, we assume that the transmitting powers of the selected source node and the relay node are fixed, i.e.,  $P_{s_i}(t) = P_{s_i}$  and  $P_{d_i}(t) = P_{d_i}$ .

1) *Max-Min Scheduling Scheme*: This user scheduling scheme is mainly designed for the FD multi-user relay system without buffer [22]. In this scheme, one pair of users have to be selected simultaneously in each time slot. The messages received by the relay node need to be forwarded to the destination node immediately owing to the lack of storage capability. The scheme is presented in the following.

Max-Min scheduling scheme: The index of the selected user pair is given by

$$k = \arg \max_{i=1, \dots, N} \min\{g_{s_i}, g_{d_i}\}, \quad (6)$$

where  $k$  is the index of the selected pair. For comparison purpose, the throughput is considered as the performance

metric in this paper. The system throughput of the Max-Min scheme is given by

$$\mathcal{T}_{tra,1} \triangleq \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{i=1}^N \sum_{t=1}^T \rho_i(t) \min\{R_{s_i}(t), R_{d_i}(t)\}, \quad (7)$$

where  $\rho_i(t)$  denotes the user scheduling indicator. If the  $i$ -th pair is selected in the  $t$ -th time slot,  $\rho_i(t) = 1$ , otherwise,  $\rho_i(t) = 0$ .  $R_{s_i}(t)$  and  $R_{d_i}(t)$  denote the transmission rates of the source nodes and relay node, respectively.

For this scheduling scheme, there are mainly two disadvantages: 1) The throughput is always limited by the worse one of the  $S_i$ -R and R- $D_i$  links. Since the relay node is not equipped with a buffer, the received messages needs to be transmitted immediately. The transmission rate is limited by the minimal one of  $R_{s_i}(t)$  and  $R_{d_i}(t)$ . 2) Some user pairs with poor channel qualities have less chance to be scheduled. The pairs with poor channel qualities can be divided into two groups. In the first group, all the pairs have poor channel qualities for both the  $S_i$ -R and R- $D_i$  links. It is intuitive that they will be selected only within a few time slots. In the second group, each pair has good channel quality for only one of the  $S_i$ -R and R- $D_i$  links and poor channel qualities of the other link. This group of users will be selected only within a few time slots as well because the worse links limit the selection probability. Actually, worse situations may happen for both groups that some users under poor channel conditions will never be selected. In this case, the channel resources of the good links are wasted and the poor links become the bottleneck for improving the system performance.

2) *Max-Max Scheduling Scheme*: This scheme is originally designed to solve the relay selection problem for the buffer-aided multi-relay system [25]. However, it could be extended to the multi-user scheduling for the considered multi-user buffer-aided relay system. For the Max-Max scheduling scheme, owing to the buffer capability, the relay node can always choose the user with best channel condition. It is described in the following.

Max-Max Scheduling Scheme: the indexes of the selected source and destination are given by

$$k_s = \arg \max_{i=1, \dots, N} \{g_{s_i}\}, \quad (8)$$

$$k_d = \arg \max_{i=1, \dots, N} \{g_{d_i}\}, \quad (9)$$

where  $k_s$  and  $k_d$  denote the indexes of the selected source and destination, respectively. For this scheduling scheme, the throughput of the system is given by

$$\mathcal{T}_{tra,2} = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{i=1}^N \min\left\{\sum_{t=1}^T p_i(t) R_{s_i}(t), \sum_{t=1}^T q_i(t) R_{d_i}(t)\right\}, \quad (10)$$

where  $p_i(t)$  and  $q_i(t)$  denote the selection indicator of the source nodes and destination nodes, respectively. If the  $i$ -th source (or destination) node is selected in the  $t$ -th time slot,  $p_i(t) = 1$  (or  $q_i(t) = 1$ ), otherwise,  $p_i(t) = 0$  (or  $q_i(t) = 0$ ).

Although the Max-Max scheme can fully utilize the wireless channel diversity, we note that it does not solve the channel unbalance problem, i.e., system performance degradation

incurred by the disparities of channel qualities between different links. The links with poor channel conditions will always have little chance to be selected. Hence, the throughput of each pair is still limited by the worse one of the  $S_i$ -R link and R- $D_i$  link. In this scheme, the same disadvantages as that of the Max-Min scheme still exist although the relay node is equipped with a buffer.

### B. Problem Formulation

To overcome the disadvantages incurred by the channel unbalance problem, in this part, we formulate the throughput maximization problem for fixed power transmission scenario. We use the binary variables  $p_i(t)$  and  $q_i(t)$  to denote the indicator of which  $S_i$  and  $D_i$  are selected in the  $t$ -th time slot, respectively. We have the following definition:

*Definition 1: For the source nodes*

$$p_i(t) = \begin{cases} 1, & \text{the node } S_i \text{ is selected} \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

*For the destination nodes*

$$q_i(t) = \begin{cases} 1, & \text{the node } D_i \text{ is selected} \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

Since we have considered the FD enabled relay node, one source node and one destination node can be selected simultaneously in each time slot. We have the following constraint.

$$\sum_{i=1}^N p_i(t) = 1, \quad (13)$$

$$\sum_{i=1}^N q_i(t) = 1. \quad (14)$$

In addition, we assume that all the source nodes have enough backlogged messages to transmit [24], [36]. Thus, the average transmission rate of the source  $S_i$  is given by

$$\bar{R}_{s_i} = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T p_i(t) R_{s_i}(t), \quad (15)$$

and the average departure rate of the  $i$ -th buffer  $B_i$  at the relay node is given by

$$\bar{R}_{d_i} = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T q_i(t) R_{d_i}(t). \quad (16)$$

The system throughput can be explained as the sum of average receiving rate of all the destination nodes. It is given by

$$\mathcal{T}_{opt} = \sum_{i=1}^N \bar{R}_{d_i} = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{i=1}^N \sum_{t=1}^T q_i(t) R_{d_i}(t). \quad (17)$$

Based on (15) and (16), the stationary conditions of the buffer at the relay node are given by *Lemma 1* and *Lemma 2*.

*Lemma 1: For a stationary buffer-aided relay system, the average arrival rate and the departure rate of the buffer  $B_i$  have to satisfy*

$$\bar{R}_{s_i} \leq \bar{R}_{d_i}, \quad i \in \{1, \dots, N\}. \quad (18)$$

*Proof:* In order to be stationary, the length of the queue in the buffer cannot increase infinitely. However, according to the queueing theory [37], if  $R_{s_i} > R_{d_i}$ , the length of the queue will approach to infinity over the infinite time horizon, which will lead to the overflow of the buffer. Hence, we have this lemma.  $\square$

*Lemma 2: To maximize the system throughput, the buffers at the relay node have to be at the edge of non-absorbing state, i.e.,*

$$\bar{R}_{s_i} = \bar{R}_{d_i}, \quad i \in \{1, \dots, N\}. \quad (19)$$

*Proof:* This is a necessary condition of the system throughput maximization. According the information flow, we have that  $\bar{R}_{s_i} \geq \bar{R}_{d_i}$ . Combing with *Lemma 1*, we have that when the throughput is maximized  $\bar{R}_{s_i} = \bar{R}_{d_i}$ .  $\square$

With this lemma and assuming the buffer size is large enough, we can observe that the minimal limitation in (3) and (4) can be neglected. When the average arrival rate equals to the average departure rate, the queue length will be in a steady state and large enough, which means the relay node always has enough messages stored in the buffers. This can also be explained as that there are only countable time slots that the buffers have insufficient messages to transmit. After being averaged over infinite time horizon, the effects of these countable time slots can be neglected [24]. Hence, we can simplify the system throughput as

$$\mathcal{T}_{opt} = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{i=1}^N \sum_{t=1}^T q_i(t) C_{d_i}(t). \quad (20)$$

Now, we are ready to present the throughput maximization problem  $\mathcal{P}_1$

$$\begin{aligned} \max_{p_i(t), q_i(t)} \quad & \mathcal{T}_{opt} \\ \text{s.t.} \quad & \bar{R}_{s_i} = \bar{R}_{d_i}, \quad \forall i \end{aligned} \quad (21)$$

$$\sum_{i=1}^N p_i(t) = 1, \quad \forall t \quad (22)$$

$$\sum_{i=1}^N q_i(t) = 1, \quad \forall t \quad (23)$$

$$p_i(t) \in \{0, 1\}, \quad \forall i, t \quad (24)$$

$$q_i(t) \in \{0, 1\}, \quad \forall i, t \quad (25)$$

where (21) ensures that every buffer is at the edge of non-absorbing state. (22) and (23) guarantee that only one source and one destination are selected during each time slot. (24) and (25) ensure that there are only two states of each source node and each destination node, i.e., selected or unselected.

We note that the original problem  $\mathcal{P}_1$  is a  $2N$  dimensional binary integer programming problem over infinite time horizon. To solve this problem, we relax the binary decision variables  $p_i(t)$  and  $q_i(t)$  to be continuous ones ranging from 0 to 1, i.e.,  $p_i(t), q_i(t) \in [0, 1]$  [38]. The problem after

relaxation is denoted by  $\mathcal{P}_2$ , which is given by

$$\begin{aligned} & \max_{p_i(t), q_i(t)} \mathcal{T}_{opt} \\ & \text{s.t. (21), (22), (23),} \\ & 1 - p_i(t) \geq 0, \quad \forall i, t \quad (26) \\ & p_i(t) \geq 0, \quad \forall i, t \quad (27) \\ & 1 - q_i(t) \geq 0, \quad \forall i, t \quad (28) \\ & q_i(t) \geq 0, \quad \forall i, t \quad (29) \end{aligned}$$

where the constraints (26) and (27) denote the range of  $p_i(t)$  after relaxation. (28) and (29) denote the range of  $q_i(t)$  after relaxation.

We note that after relaxation, the original problem converts to a  $2N$ -dimensional linear programming problem over the infinite time horizon. For this problem, the optimal solution locates at the boundary of the feasible set, i.e.,  $p_i(t), q_i(t) = 0$  or  $1$ , which means that the problems  $\mathcal{P}_1$  and  $\mathcal{P}_2$  have the same solution. Hence, the optimal solution of the problem  $\mathcal{P}_2$  is also the optimal solution of the original problem, which will be shown in the proof of *Theorem 1*.

### C. Optimal Multi-User Scheduling Scheme With Fixed Power Transmission

In this section, we present the proposed optimal multi-user scheduling scheme for the fixed power transmission scenario of the considered system.

*Theorem 1: The proposed optimal multi-user scheduling scheme for the considered multi-user buffer-aided FD relay system under fixed power transmission scenario is given by:*

*For the source nodes:*

$$p_k(t) = \begin{cases} 1, & k = \arg \max_{i=1, \dots, N} \Gamma_i(t) \\ 0, & \text{otherwise} \end{cases} \quad (30)$$

*For the destination nodes:*

$$q_k(t) = \begin{cases} 1, & k = \arg \max_{i=1, \dots, N} \Lambda_i(t) \\ 0, & \text{otherwise} \end{cases} \quad (31)$$

where the selection functions are given by

$$\Gamma_i(t) = -\lambda_i, \quad (32)$$

$$\Lambda_i(t) = (1 + \lambda_i)\mathcal{F}(\gamma_{d_i}(t)), \quad (33)$$

where

$$\mathcal{F}(x) = \log_2(1 + x), \quad (34)$$

and  $\lambda_i \in (-1, 0)$  denotes the optimal weight factor of the  $i$ -th pair and it satisfies the following equation

$$\mathbb{E}\{p_i(t)C_{s_i}(t)\} = \mathbb{E}\{q_i(t)C_{d_i}(t)\}. \quad (35)$$

*Proof:* Please see Appendix A.  $\square$

It is noted that this scheme is particularly suitable for the i.n.i.d. fading environment. Unlike the traditional multi-user scheduling schemes, in which the system performance is limited by the worse links, the proposed scheme can balance the disparities between the  $S_i$ -R and R- $D_i$  links by introducing the weight factor  $\lambda_i$ . For instance, we assume that the average

channel gains of the first pair are 0.9 and  $\Omega_{d_1} = 0.1$ . We may set  $\lambda_1 = -0.1$ , then  $1 + \lambda_1 = 0.9$ , which is equivalent to that the  $S_1$ -R link is weakened by factor 0.1 but the R- $D_1$  link is only weakened with factor 0.9 (see Fig. 2 and Fig. 4 for more details). The source node  $S_1$  will be selected in less time slots and the destination node  $D_1$  will be selected in more time slots. By this way, the throughput of the link  $S_1$ -R and link R- $D_i$  can be balanced.

In the proposed optimal multi-user scheduling scheme, the optimal weight factor  $\lambda_i$  of each pair is given by the following corollary.

*Corollary 1: For the proposed optimal multi-user scheduling scheme in Theorem 1, the optimal weight factor  $\lambda_i$  can be obtained by solving the following equation*

$$\begin{aligned} & \int_0^\infty \left[ \prod_{j=1, j \neq i}^N \int_0^{\mathcal{H}_{s_j}^i} f_{\gamma_{s_j}}(x_j) dx_j \right] \log_2(1 + x_i) f_{\gamma_{s_i}}(x_i) dx_i \\ & = \int_0^\infty \left[ \prod_{j=1, j \neq i}^N \int_0^{\mathcal{H}_{d_j}^i} f_{\gamma_{d_j}}(y_j) dy_j \right] \log_2(1 + y_i) f_{\gamma_{d_i}}(y_i) dy_i, \end{aligned} \quad (36)$$

where

$$\mathcal{H}_{s_j}^i = \mathcal{F}^{-1}\left(\frac{\lambda_i}{\lambda_j} \mathcal{F}(x_i)\right), \quad (37)$$

$$\mathcal{H}_{d_j}^i = \mathcal{F}^{-1}\left(\frac{1 + \lambda_i}{1 + \lambda_j} \mathcal{F}(y_i)\right), \quad (38)$$

and  $f_{\gamma_{s_i}}(x_i)$ ,  $f_{\gamma_{d_i}}(y_i)$  denote the probability density functions (pdfs) of the SINRs  $\gamma_{s_i}$  and  $\gamma_{d_i}$ , respectively.  $\mathcal{F}^{-1}(x)$  denotes the inverse function of  $\mathcal{F}(x)$  in (34).

*Proof:* Please see Appendix B.  $\square$

With this corollary, we can obtain the optimal weight factor  $\lambda_i$  using the built-in functions of the software packages such as Matlab or Mathematica. Based on the optimal multi-user scheduling scheme, the system performance is given by the following corollary.

*Corollary 2: With the optimal multi-user scheduling scheme, the system throughput can be evaluated by*

$$\mathcal{T}_{opt} = \sum_{i=1}^N \int_0^\infty \left[ \prod_{j=1, j \neq i}^N \int_0^{\mathcal{H}_{d_j}^i} f_{\gamma_{d_j}}(y) dy \right] \log_2(1 + x) f_{\gamma_{d_i}}(x) dx. \quad (39)$$

*Proof:* Referring to the Appendix B and then using the Total Probability Theorem, this corollary can be obtained. The details of the proof are omitted here.  $\square$

## IV. OPTIMAL USER SCHEDULING SCHEME WITH ADAPTIVE POWER TRANSMISSION

In addition to the scheme for the fixed power transmission scenario, in this section, we propose an optimal multi-user scheduling scheme under adaptive power transmission scenario. We consider the system average sum power constraint, under which the transmit powers of the source nodes and the relay node are adaptively designed to maximize the system throughput.

### A. Problem Formulation

The problem of the optimal multi-user scheduling scheme with adaptive power transmission is formulated in this section. The average power consumption of each source node and the relay node forwarding information to each destination is given by

$$\bar{P}_{s_i} = \mathbb{E}\{p_i(t)P_{s_i}(t)\} = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T p_i(t)P_{s_i}(t), \quad (40)$$

$$\bar{P}_{d_i} = \mathbb{E}\{q_i(t)P_{d_i}(t)\} = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T q_i(t)P_{d_i}(t), \quad (41)$$

respectively.

We consider the following the sum power consumption constraint

$$\sum_{i=1}^N \bar{P}_{s_i} + \sum_{i=1}^N \bar{P}_{d_i} \leq \bar{P}_{sum}. \quad (42)$$

The throughput maximization problem  $\mathcal{P}_3$  under the average sum power constraint after relaxation [38] is given by

$$\begin{aligned} & \max_{p_i(t), q_i(t), \varphi_{s_i}(t), \varphi_{d_i}(t)} \mathcal{T}_{opt} \\ & \text{s.t. (21), (22), (23), (26), (27), (28), (29)} \end{aligned} \quad (28), (29)$$

$$\sum_{i=1}^N \bar{P}_{s_i} + \sum_{i=1}^N \bar{P}_{d_i} \leq \bar{P}_{sum} \quad (43)$$

$$P_{s_i}(t) \geq 0, \quad \forall i \quad (44)$$

$$P_{d_i}(t) \geq 0, \quad \forall i \quad (45)$$

### B. Optimal Multi-User Scheduling Scheme With Adaptive Power Transmission

In this section, we propose an optimal multi-user scheduling scheme for the considered adaptive power transmission scenario.

*Theorem 2: The optimal multi-user scheduling scheme with adaptive power allocation is given by*

$$p_k(t) = \begin{cases} 1, & k = \arg \max_i \Psi_i(t) \\ 0, & \text{otherwise} \end{cases} \quad (46)$$

$$q_k(t) = \begin{cases} 1, & k = \arg \max_i \Phi_i(t) \\ 0, & \text{otherwise} \end{cases} \quad (47)$$

$$P_{s_k}(t) = \begin{cases} \frac{-\mu_k}{\ln(2)\nu} - \frac{1}{\bar{g}_{s_k}(t)}, & \text{if } \bar{g}_{s_k}(t) > \frac{\ln(2)\nu}{-\mu_k} \\ 0, & \text{otherwise} \end{cases} \quad (48)$$

$$P_{d_k}(t) = \begin{cases} \frac{1+\mu_k}{\ln(2)\nu} - \frac{1}{\bar{g}_{d_k}(t)}, & \text{if } \bar{g}_{d_k}(t) > \frac{\ln(2)\nu}{1+\mu_k} \\ 0, & \text{otherwise} \end{cases} \quad (49)$$

where selection functions  $\Psi_i(t)$  and  $\Phi_i(t)$  are given by

$$\Psi_i(t) = -\mu_i \log_2[1 + P_{s_i}(t)\bar{g}_{s_i}(t)] - \nu P_{s_i}(t), \quad (50)$$

$$\Phi_i(t) = (1 + \mu_i) \log_2[1 + P_{d_i}(t)\bar{g}_{d_i}(t)] - \nu P_{d_i}(t), \quad (51)$$

where  $\mu_i$  and  $\nu$  denote the Lagrangian multipliers.

*Proof:* Please refer to Appendix C.  $\square$

From the above power allocation expressions, it is noted that the transmit power may equal to zero when the channel gain is below a certain threshold. This means that when a source node or destination node is selected, it may stay silent in these time slots due to the over low instantaneous channel gain. This phenomenon coincides with our intuition that these time slots will consume too much power but not bring much performance improvement. In this case, it is better to keep silent and use the saved power to transmit messages when the instantaneous channel becomes more moderate.

The optimal value of the multipliers  $\mu_i$  and  $\nu$  can be found by the following corollary.

*Corollary 3: The optimal value of  $\mu_i$  and  $\nu$  can be obtained by solving the following equations*

$$\begin{aligned} & \int_{-\frac{\ln(2)\nu}{\mu_i}}^{\infty} \left[ \prod_{j=1, j \neq i}^M \zeta_{s_j}^i \right] \log_2 \left( \frac{-\mu_i y}{\ln(2)\nu} \right) f_{\bar{g}_{s_i}}(x_i) dx_i \\ & = \int_{\frac{\ln(2)\nu}{(1+\mu_i)}}^{\infty} \left[ \prod_{j=1, j \neq i}^M \zeta_{d_j}^i \right] \log_2 \left( \frac{(1+\mu_i)y_i}{\ln(2)\nu} \right) f_{\bar{g}_{d_i}}(y_i) dy_i, \end{aligned} \quad (52)$$

$$\begin{aligned} & \sum_{i=1}^M \left\{ \int_{-\frac{\ln(2)\nu}{\mu_i}}^{\infty} \left[ \prod_{j=1, j \neq i}^M \zeta_{s_j}^i \right] \left( \frac{-\mu_i}{\ln(2)\nu} - \frac{1}{x_i} \right) f_{\bar{g}_{s_i}}(x_i) dx_i \right. \\ & \left. + \int_{\frac{\ln(2)\nu}{(1+\mu_i)}}^{\infty} \left[ \prod_{j=1, j \neq i}^M \zeta_{d_j}^i \right] \left( \frac{1+\mu_i}{\ln(2)\nu} - \frac{1}{y_i} \right) f_{\bar{g}_{d_i}}(y_i) dy_i \right\} = \bar{P}_{sum}, \end{aligned} \quad (53)$$

where

$$\zeta_{s_j}^i = \int_0^{\xi_{s_j}^i} f_{\bar{g}_{s_j}}(x_j) dx_j, \quad (54)$$

$$\zeta_{d_j}^i = \int_0^{\xi_{d_j}^i} f_{\bar{g}_{d_j}}(y_j) dy_j, \quad (55)$$

$$\xi_{s_j}^i = \frac{\ln(2)\nu}{\mu_j \mathcal{W} \left( -\exp \left( \frac{\mu_i}{\mu_j} + \frac{\ln(2)\nu}{\mu_j x_i} - 1 \right) \left( \frac{\ln(2)\nu}{-\mu_i x_i} \right)^{\frac{\mu_i}{\mu_j}} \right)}, \quad (56)$$

$$\xi_{d_j}^i = \frac{-\ln(2)\nu / (1 + \mu_j)}{\mathcal{W} \left( -\exp \left( \frac{1+\mu_i}{1+\mu_j} - \frac{\ln(2)\nu}{(1+\mu_j)y_i} - 1 \right) \left( \frac{\ln(2)\nu}{(1+\mu_i)y_i} \right)^{\frac{1+\mu_i}{1+\mu_j}} \right)}, \quad (57)$$

where  $\mathcal{W}(\cdot)$  denotes the Lambert W-function.

*Proof:* Please see Appendix D.  $\square$

It is noted that for the optimal  $\mu_i$  and  $\nu$ , they has to satisfy  $-1 < \mu_i < 0$  and  $\nu > 0$ . To obtain the optimal  $\mu_i$  and  $\nu$ , we have to solve the equations (52) and (53). These two functions can be solved by the numerical methods using softwares such as Matlab and Mathematica. Actually, since (52) and (53) only depends on the statistical characteristics of  $\bar{g}_{s_i}(t)$  and  $\bar{g}_{d_i}(t)$ , the optimal value of  $\mu_i$  and  $\nu$  can be obtained in advance and be used for the subsequent time slots.

*Corollary 4: With the optimal user scheduling and adaptive power transmission, the system throughput can be evaluated by*

$$T_{opt} = \sum_{i=1}^N \int_{\frac{\ln(2)\nu}{(1+\mu_i)}}^{\infty} \left[ \prod_{j=1, j \neq i}^N \zeta_{d_j}^i \right] \log_2 \left( \frac{(1+\mu_i)y_i}{\ln(2)\nu} \right) f_{\bar{g}_{d_i}}(y_i) dy_i. \quad (58)$$

*Proof:* Referring to the Appendix D and using the Total Probability Theorem, the system throughput can be obtained.  $\square$

## V. PERFORMANCE ANALYSIS

In this section, we consider the practical Rayleigh fading environment and present specific system throughput expressions of the proposed optimal scheduling schemes with the fixed power transmission and adaptive power transmission.

### A. System Throughput With Fixed Power Transmission

We consider the Rayleigh distributed channel gains to model the non-line-of-sight propagation environment. Hence, the link SINRs  $\gamma_{s_i}(t)$  and  $\gamma_{d_i}(t)$  are subject to the exponential distribution. The *pdfs* of  $f_{\gamma_{s_i}}(x)$  and  $f_{\gamma_{d_i}}(y)$  are given by

$$f_{\gamma_{s_i}}(x_i) = \frac{1}{\Theta_{s_i}} \exp\left(-\frac{x_i}{\Theta_{s_i}}\right), \quad (59)$$

$$f_{\gamma_{d_i}}(y_i) = \frac{1}{\Theta_{d_i}} \exp\left(-\frac{y_i}{\Theta_{d_i}}\right), \quad (60)$$

where  $\Theta_{s_i}$  and  $\Theta_{d_i}$  denote the mean values of the random variables  $\gamma_{s_i}(t)$  and  $\gamma_{d_i}(t)$ , respectively.

Then, (37) and (38) can be simplified as

$$\mathcal{H}_{s_j}^i = (1 + x_i)^{\frac{\lambda_j}{\lambda_i}} - 1, \quad (61)$$

$$\mathcal{H}_{d_j}^i = (1 + y_i)^{\frac{1+\lambda_j}{1+\lambda_i}} - 1. \quad (62)$$

After some simple operations, (36) can be simplified as

$$\begin{aligned} & \int_0^\infty \left[ \prod_{j=1, j \neq i}^M \left( 1 - \exp\left(-\frac{(1+x_i)^{\frac{\lambda_j}{\lambda_i}} - 1}{\Theta_{s_j}}\right) \right) \right] \\ & \times \log_2(1+x_i) \frac{1}{\Theta_{s_i}} \exp\left(-\frac{x_i}{\Theta_{s_i}}\right) dx_i \\ & = \int_0^\infty \left[ \prod_{j=1, j \neq i}^M \left( 1 - \exp\left(-\frac{(1+x_i)^{\frac{1+\lambda_j}{1+\lambda_i}} - 1}{\Theta_{d_j}}\right) \right) \right] \\ & \times \log_2(1+x_i) \frac{1}{\Theta_{d_i}} \exp\left(-\frac{x_i}{\Theta_{d_i}}\right) dx_i, \end{aligned} \quad (63)$$

by which we can find the optimal  $\lambda_i$  via numerical methods. Then the optimal system throughput of (39) is given by

$$\begin{aligned} \mathcal{T}_{opt} &= \sum_{i=1}^N \int_0^\infty \log_2(1+x_i) \frac{1}{\Theta_{d_i}} \exp\left(-\frac{x_i}{\Theta_{d_i}}\right) \\ & \times \left[ \prod_{j=1, j \neq i}^M \left( 1 - \exp\left(-\frac{(1+x_i)^{\frac{1+\lambda_j}{1+\lambda_i}} - 1}{\Theta_{d_j}}\right) \right) \right] dx_i. \end{aligned} \quad (64)$$

### B. System Throughput With Adaptive Power Transmission

Under the consideration of Rayleigh distribution, the distribution of  $\bar{g}_{s_i}(t)$  and  $\bar{g}_{d_i}(t)$  are given by

$$f_{g_{s_i}}(x_i) = \frac{1}{\bar{\Omega}_{s_i}} \exp\left(-\frac{x_i}{\bar{\Omega}_{s_i}}\right), \quad (65)$$

$$f_{g_{d_i}}(y_i) = \frac{1}{\bar{\Omega}_{d_i}} \exp\left(-\frac{y_i}{\bar{\Omega}_{d_i}}\right). \quad (66)$$

Accordingly, (52) and (53) can be expressed as

$$\begin{aligned} & \int_{-\frac{\ln(2)\nu}{\mu_i}}^\infty \left[ \prod_{j=1, j \neq i}^M (1 - \exp(\frac{\xi_{s_j}^i}{\Omega_{s_j}})) \right] \log_2\left(\frac{-\mu_i x_i}{\ln(2)\nu}\right) \frac{\exp(\frac{-x_i}{\Omega_{s_i}})}{\Omega_{s_i}} dx_i \\ & = \int_{\frac{\ln(2)\nu}{(1+\mu_i)}}^\infty \left[ \prod_{j=1, j \neq i}^M (1 - \exp(\frac{\xi_{d_j}^i}{\Omega_{d_j}})) \right] \log_2\left(\frac{(1+\mu_i)y_i}{\ln(2)\nu}\right) \frac{\exp(\frac{-y_i}{\Omega_{d_i}})}{\Omega_{d_i}} dy_i, \end{aligned} \quad (67)$$

$$\begin{aligned} & \sum_{i=1}^M \left\{ \int_{-\frac{\ln(2)\nu}{\mu_i}}^\infty \left[ \prod_{j=1, j \neq i}^M (1 - \exp(\frac{\xi_{s_j}^i}{\Omega_{s_j}})) \right] \right. \\ & \times \left(\frac{-\mu_i}{\ln(2)\nu} - \frac{1}{x_i}\right) \frac{\exp(\frac{-x_i}{\Omega_{s_i}})}{\Theta_{s_i}} dx_i \\ & \left. + \int_{\frac{\ln(2)\nu}{(1+\mu_i)}}^\infty \left[ \prod_{j=1, j \neq i}^M (1 - \exp(\frac{\xi_{d_j}^i}{\Omega_{d_j}})) \right] \right. \\ & \left. \times \left(\frac{1+\mu_i}{\ln(2)\nu} - \frac{1}{y_i}\right) \frac{\exp(\frac{-y_i}{\Omega_{d_i}})}{\Omega_{d_i}} dy_i \right\} = \bar{P}_{sum}, \end{aligned} \quad (68)$$

where  $\xi_{1j}^i$  and  $\xi_{2j}^i$  are given by (56) and (57).

Based on the two equations, we can obtain the optimal Lagrangian multipliers  $\mu_i$  and  $\nu$ . Then, the system throughput can be evaluated by

$$\begin{aligned} \mathcal{T}_{opt} &= \sum_{i=1}^M \int_{\frac{\ln(2)\nu}{(1+\mu_i)}}^\infty \left[ \prod_{j=1, j \neq i}^M (1 - \exp(\frac{\xi_{d_j}^i}{\Omega_{d_j}})) \right] \\ & \times \log_2\left(\frac{(1+\mu_i)y_i}{\ln(2)\nu}\right) \frac{\exp(\frac{-y_i}{\Omega_{d_i}})}{\Omega_{d_i}} dy_i. \end{aligned} \quad (69)$$

## VI. NUMERICAL SIMULATIONS

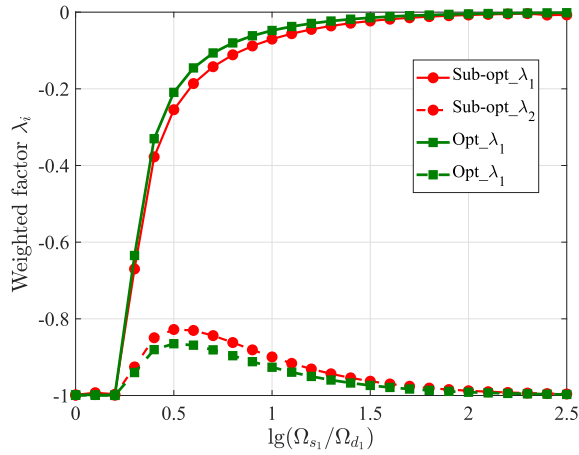
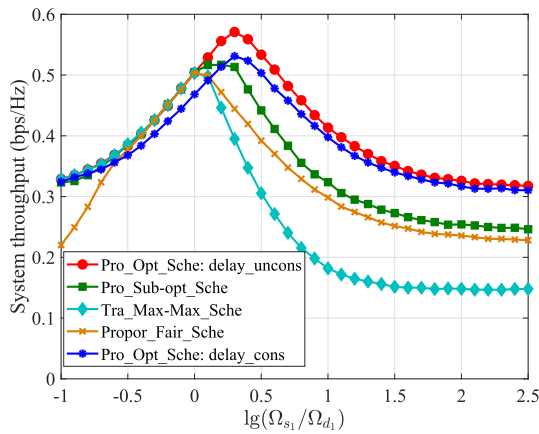
In this section, theoretical evaluations and Monte Carlo (MC) simulations are presented to illustrate the system throughput of the proposed multi-user scheduling schemes. The throughput of the Max-Max scheduling scheme [25] and the PFS scheme [27], [28] are presented as well. It is noted that the simulation results in [25] have shown that the Max-Max scheme outperforms the Max-Min scheme. Hence, we only compare the Max-Max scheme with the proposed schemes in terms of the system throughput.

### A. Fixed Power Transmission

In this section, we consider two cases: i) changing the qualities of the channels between the source and relay, relay and destination; ii) changing the qualities of the channels between different source nodes and the relay node. In addition, we present the sub-optimal method, in which we replace the optimal metric function  $\mathcal{F}(x) = \log(1+x)$  in the equation (34) with  $\tilde{\mathcal{F}}(x) = x$ , which achieves the similar system performance but reduces the computation complexity.

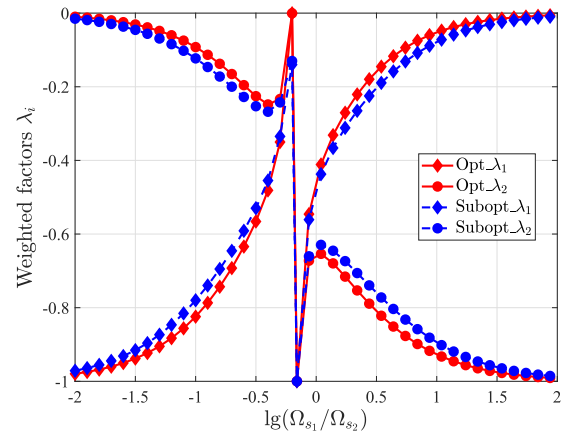
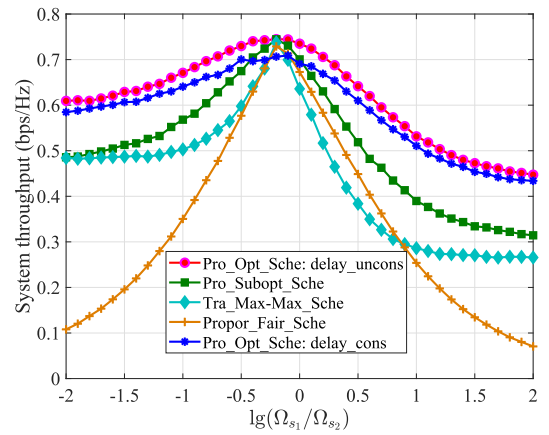
Fig. 2 plots the optimal and sub-optimal weight factor  $\lambda_i$  for the case of changing the channel qualities of the  $S_1$ -R and R- $D_1$  links. We set  $N = 2$ ,  $\Omega_{s_2} = 0.4$ ,  $\Omega_{d_2} = 0.6$  and  $\Omega_{s_1} + \Omega_{d_1} = 1$ . The variances of the noise and the RSI are set to 1 [18]. By changing the ratio of  $\Omega_{s_1}/\Omega_{d_1}$ ,



Fig. 2. Weight factors with changing the qualities of  $S_1$ -R and R- $D_1$  links.Fig. 3. System throughput with changing the qualities of  $S_1$ -R and R- $D_1$  links.

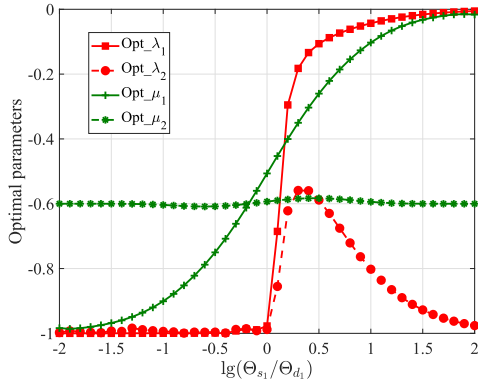
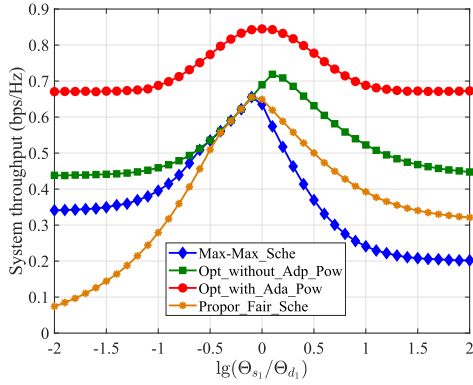
we calculate the weight factors  $\lambda_1$  and  $\lambda_2$ . The simulation results verify that the value of  $\lambda_i$  varies between  $-1$  and  $0$ . In addition, we note that with the increase of the ratio  $\Omega_{s_1}/\Omega_{d_1}$ ,  $\lambda_1$  increases from  $-1$  to  $0$ , which is consistent with our intuition, i.e., the larger  $\lambda_1$  leads to that  $S_1$  is selected in less time slots than  $D_1$ . What's more, the variation of  $\Omega_{s_1}/\Omega_{d_s}$  will also affect the value of  $\lambda_2$ . When  $\Omega_{s_1}/\Omega_{d_1}$  is very large,  $\lambda_2$  will approach to  $-1$ , which implies that  $S_1$  is selected in less time slots and  $S_2$  is selected in more time slots. This insight reveals the advantages of our proposed multi-user scheduling scheme. By multiplying different weight factors, we can adjust the selection probabilities of different nodes. In this way, the performance gaps between different links can be balanced.

Fig. 3 shows the system throughput with the variation of the ratio of  $\Omega_{s_1}/\Omega_{d_1}$ . In this figure, we compare the system throughput of the proposed scheme with that of the Max-Max scheme and the PFS scheme. In addition, the system throughput of proposed scheme without delay constraint is compared to that under delay constraint, where we consider the ‘‘buffer-starvation’’ policy, i.e., when the latency of the packets in the buffer is close to the delay constraint, the corresponding relay-to-destination link will be scheduled in the following time slot. This scheme can only serve as a heuristic policy to

Fig. 4. Weight factors with changing the qualities of  $S_1$ -R and  $S_2$ -R links.Fig. 5. System throughput with changing the qualities of  $S_1$ -R and  $S_2$ -R links.

deal with the delay constraint. In the numerical simulation, the delay constraint is 60 for each pair. Once the latency of the packets in one buffer is over 50, the corresponding destination node will be selected in the following time slot. This figure demonstrates the superiority of the proposed user scheduling scheme, which achieves better performance than both the Max-Max scheduling scheme and the PFS scheme. The results validate that the PFS scheme does not fit for the two-hop relay systems. In addition, it is noted that system with delay constraint almost achieves the same throughput as that without delay constraint. However, within some region, all the scheduling schemes achieve the same performance. The reason is that all the source to relay links almost have the same quality and all the relay-to-destination almost have the same quality. Hence, the proposed scheme degrades to the Max-Max scheduling scheme.

In Fig. 4, we plot the optimal and sub-optimal weight factor  $\lambda_i$  while changing the channel qualities of  $S_1$ -R and  $S_2$ -R links. We set  $\Omega_{d_1} = 0.4$ ,  $\Omega_{d_2} = 0.6$  and  $\Omega_{s_1} + \Omega_{s_2} = 1$  and change the ratio of  $\Omega_{s_1}/\Omega_{s_2}$ . The variances of the RSI and noise are set to 1. This figure shows the relationship between the variation of the weight factors  $\lambda_1$  and  $\lambda_2$ . The results reveal that in the low  $\Omega_{s_1}/\Omega_{s_2}$  regime, i.e., the  $S_1$ -R link is weaker than the  $S_2$ -R link,  $\lambda_1$  is close to  $-1$ , but  $\lambda_2$


 (a) Optimal  $\lambda_i$  and  $\mu_i$ 


(b) System throughput

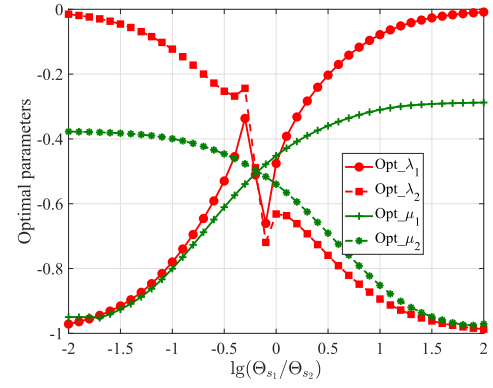
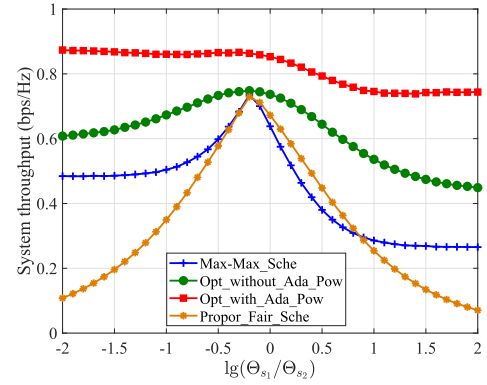
 Fig. 6. The optimal weight factors and the corresponding system throughput with changing the qualities of the  $S_1$ -R and R- $D_1$  links.

is close to 0. Hence, the  $S_1$ -R link will be selected in more time slots than the  $S_2$ -R link. This insight reveals another advantage of the proposed scheme, i.e., the ability to balance the throughput gaps between different source-to-relay links. In addition, we note that when  $\Omega_{s_1}/\Omega_{s_2} = 2/3$ ,  $\lambda_1$  and  $\lambda_2$  almost have the same value and the curves are almost vertical. In this case, the links  $S_1$ -R and  $S_2$ -R have the same quality, the links R- $D_1$  and R- $D_2$  have the same quality. The proposed scheme degrades to the Max-Max user scheduling scheme.

Fig. 5 plots the system throughput with the variation of the ratio  $\Omega_{s_1}/\Omega_{s_2}$ . The parameters are set the same as that of Fig. 4. Fig. 5 demonstrates the superiorities of the proposed scheme, i.e., achieving maximum system throughput. For the PFS scheme, the throughput decreases rapidly in the highly unbalanced regime, which validates that PFS scheme cannot serve the multi-user relay systems well. By designing the weight factors, the proposed scheme always achieves higher throughput. In addition, we note that when  $\Omega_{s_1}/\Omega_{s_2} = 2/3$ , all the schemes achieve the same system throughput, meaning that the proposed scheme degrades to the Max-Max scheme.

### B. Adaptive Power Transmission

In this section, we evaluate the system performance of the proposed scheme with adaptive power transmission. The transmit power is adaptively designed under the total average


 (a) Optimal  $\lambda_i$  and  $\mu_i$ 


(b) System throughput

 Fig. 7. The optimal weight factors and the system throughput with changing the qualities of  $S_1$ -R and  $S_2$ -R links.

power consumption constraint. We compare the performance of the proposed scheme under the fixed power transmission, Max-Max scheme and the PFS scheme.

Fig. 6 investigates the system performance of the proposed multi-user scheduling scheme with changing the channel qualities of the  $S_1$ -R and R- $D_1$  links. Fig. 6(a) shows the obtained optimal weight factors  $\lambda_i$  and  $\mu_i$ . Fig. 6(b) compares the corresponding system throughput under different schemes. In the simulation, we set  $\Theta_{s_2} = 0.4$ ,  $\Theta_{d_2} = 0.6$  and  $\Theta_{s_1} + \Theta_{d_1} = 1$ . The total average power is set as  $P_{sum} = 2$ . This figure reveals that the system throughput of the adaptive power transmission scenario outperforms that of the fixed power transmission scenario. In addition, the proposed scheme for adaptive power transmission always achieves higher throughput than both the Max-Max and PFS scheme.

Fig. 7 illustrates the system throughput of the proposed scheme with adaptive power transmission by changing of channel qualities of the  $S_1$ -R and  $S_2$ -R links. Fig. 7(a) shows the obtained optimal weight factor  $\lambda_i$  and  $\mu_i$ . The system throughput under different schemes is presented in Fig. 7(b). The parameters are set as  $\Theta_{d_1} = 0.4$ ,  $\Theta_{d_2} = 0.6$  and  $\Theta_{s_1} + \Theta_{s_2} = 1$ . This figure exhibits that the adaptive power transmission scenario outperforms the fixed power transmission scenario, as well as the Max-Max scheme and the PFS scheme. The results further confirm the superiority of the proposed scheme and the proposed power allocation. By designing the

optimal weight factors according to the statistical CSI and combining with the adaptive power transmission, the performance gaps between different links can be alleviated and the system throughput can be improved.

## VII. CONCLUSION

In this paper, we addressed the channel unbalance problem of the i.n.i.d. multi-user scheduling relay system. In particular, two optimal scheduling schemes were proposed for FD buffer-aided multi-user relay system under the fixed and adaptive power transmission scenario, respectively. In the proposed schemes, the weight factor of each pair was calculated according to the statistical CSI. By designing the optimal weight factors, the proposed schemes could mitigate the throughput gaps between different links. In addition, in the adaptive power transmission scenario, the closed-form optimal power allocation expressions were derived. Combining the optimal weight factors with optimal power allocation expressions, novel selection functions were obtained. Numerical simulations verified superiority in terms of achieving maximum system throughput of the proposed schemes.

### APPENDIX A PROOF OF THEOREM 1

We note that the problem  $\mathcal{P}_2$  is a  $2N$ -dimensional linear programming problem over infinite time horizon. For this problem, the optimal solution satisfies the KKT optimal conditions. Hence, the Lagrangian function can be expressed as

$$\begin{aligned} \mathcal{L}(p_i(t), q_i(t), \lambda_i, \alpha(t), \beta(t), \mu_i(t), \nu_i(t), \varepsilon_i(t), \eta_i(t)) \\ = \frac{1}{T} \sum_{i=1}^N \sum_{t=1}^T q_i(t) C_{d_i}(t) \\ + \frac{1}{T} \sum_{t=1}^T \lambda_i [q_i(t) C_{d_i}(t) - p_i(t) C_{s_i}(t)] \\ + \alpha(t) [1 - \sum_{i=1}^N p_i(t)] + \beta(t) [1 - \sum_{i=1}^N q_i(t)] \\ + \mu_i(t) [1 - p_i(t)] + \nu_i(t) p_i(t) \\ + \varepsilon_i(t) [1 - q_i(t)] + \eta_i(t) q_i(t) \end{aligned} \quad (70)$$

where  $\lambda_i$ ,  $\alpha(t)$ ,  $\beta(t)$ ,  $\mu_i(t)$ ,  $\nu_i(t)$ ,  $\varepsilon_i(t)$  and  $\eta_i(t)$  are the Lagrange multipliers. Differentiate the  $\mathcal{L}(\cdot)$  function with regard to  $p_i(t)$  and  $q_i(t)$ , and set them to zero, i.e.,

$$\frac{\partial \mathcal{L}}{\partial p_i(t)} = 0, \quad \frac{\partial \mathcal{L}}{\partial q_i(t)} = 0, \quad (71)$$

we get

$$-\lambda_i \frac{1}{T} C_{s_i}(t) - \alpha(t) - \mu_i(t) + \nu_i(t) = 0, \quad (72)$$

$$(1 + \lambda_i) \frac{1}{T} C_{d_i}(t) - \beta(t) - \varepsilon_i(t) + \eta_i(t) = 0, \quad (73)$$

respectively.

If we let  $p_k(t) = 1$ , we obtain that  $\nu_k(t) = 0$ ,  $\mu_i(t) = 0$ ,  $i \neq k$ , according to the complementary slackness theorem. Substituting them into (72), we have

$$\begin{aligned} T[\alpha(t) + \mu_k(t)] &= -\lambda_k C_{s_k}(t) \triangleq \Gamma_k(t) \\ T[\alpha(t) - \nu_i(t)] &= -\lambda_i C_{s_i}(t) \triangleq \Gamma_i(t), \quad i \neq k \end{aligned} \quad (74)$$

Since  $\mu_k(t) \geq 0$ ,  $\nu_i(t) \geq 0$ , we have  $\Gamma_k(t) \geq \Gamma_i(t)$ ,  $i \neq k$ , where  $\Gamma_i(t)$  denotes the optimal decision function for the source nodes. Here, we can conclude that if  $p_k(t) = 1$ ,  $\Gamma_k(t) \geq \max\{\Gamma_i(t)\}$ . Hitherto, we obtain the optimal scheduling scheme for the source nodes. In the similar way, if we set  $q_k(t) = 1$ , we have

$$\begin{aligned} T[\beta(t) + \varepsilon_k(t)] &= (1 + \lambda_k) C_{d_k}(t) = \Lambda_k, \\ T[\beta(t) - \eta_i(t)] &= (1 + \lambda_i) C_{d_i}(t) = \Lambda_i, \end{aligned} \quad (75)$$

where  $\varepsilon_k(t) \geq 0$ ,  $\eta_i(t) \geq 0$ , we have  $\Lambda_k(t) \geq \Lambda_i(t)$ ,  $i \neq k$ , which denotes the selection functions for the destination nodes. Hitherto, we obtain the optimal scheduling scheme for the destination nodes.

Next, we need to specify the range of the optimal weight factors  $\lambda_i$ . First, in order to make the buffers at the edge of non-absorb state, the average arrival rate should equal to the average departure rate, we have

$$\mathbb{E}\{p_i(t) C_{s_i}(t)\} = \mathbb{E}\{q_i(t) C_{d_i}(t)\}. \quad (76)$$

Second, if  $\lambda_i = 0$  or  $\lambda_i + 1 = 0$ , the corresponding source  $S_i$  or destination  $D_i$  will never be selected. Hence, we have  $\lambda_i \notin \{0, 1\}$ . Besides, we consider a special case, i.e., i.i.d. fading environment, which means all the links have the same quality. In this case,  $\lambda_1 = \lambda_2 = \dots = \lambda_N$ . In order to maximize the system throughput, the source node with the maximum  $C_{s_i}(t)$  and the destination node with maximum  $C_{d_i}(t)$  will be selected, which means  $\lambda_{k_1} < 0$  and  $1 + \lambda_{k_2} > 0$ . Hence, we can conclude that  $\lambda_i \in (-1, 0)$ .

### APPENDIX B PROOF OF COROLLARY 1

First, we define 2  $N$ -dimensional random variables,  $\mathbf{X} = \{x_1, x_2, \dots, x_N\}$  and  $\mathbf{Y}(t) = \{y_1, y_2, \dots, y_N\}$ , of which the pdfs are

$$f_{\gamma_s}(\mathbf{X}) = \prod_{i=1}^N f_{\gamma_{s_i}}(x_i), \quad (77)$$

$$f_{\gamma_d}(\mathbf{Y}) = \prod_{i=1}^N f_{\gamma_{d_i}}(y_i). \quad (78)$$

Hence, for  $i \in \{1, \dots, N\}$ , we have

$$\begin{aligned} \mathbb{E}\{p_i(t) C_{s_i}(t)\} &= \int_{-\lambda_j \mathcal{F}(\gamma_{s_j}) < -\lambda_i \mathcal{F}(\gamma_{d_i})} f_{\gamma_s}(\mathbf{X}) \log_2(1 + x_i) d\mathbf{X} \\ &= \int_0^\infty \left\{ \left[ \prod_{j=1, j \neq i}^N \int_0^{\mathcal{H}^i_{\gamma_{s_j}}} f_{\gamma_{s_j}}(x_j) dx_j \right] \right. \\ &\quad \left. \times \log_2(1 + x_i) f_{\gamma_{s_i}}(x_i) \right\}, \end{aligned} \quad (79)$$

$$\begin{aligned} \mathbb{E}\{q_i(t) C_{d_i}(t)\} &= \int_{(1+\lambda_j) \mathcal{F}(\gamma_{d_j}) < (1+\lambda_i) \mathcal{F}(\gamma_{d_i})} f_{\gamma_d}(\mathbf{Y}) \log_2(1 + y_i) d\mathbf{Y} \\ &= \int_0^\infty \left\{ \left[ \prod_{j=1, j \neq i}^N \int_0^{\mathcal{H}^i_{\gamma_{d_j}}} f_{\gamma_{d_j}}(y_j) dy_j \right] \right. \\ &\quad \left. \times \log_2(1 + y_i) f_{\gamma_{d_i}}(y_i) \right\}, \end{aligned} \quad (80)$$

where  $\mathcal{H}_{s_j}^i$  and  $\mathcal{H}_{r_j}^i$  are given by (37) and (38). It is noted that since we consider the block fading channel, the time slot indexes are neglected in the R.H.S. of (79) and (80). Substituting (79) and (80) into (35), we can get *Corollary 1*, which ends the proof.

### APPENDIX C PROOF OF THEOREM 2

To solve problem  $\mathcal{P}_3$ , we first write the Lagrangian function as

$$\begin{aligned} & \mathcal{L}(p_i(t), q_i(t), \varphi_{s_i}(t), \varphi_{d_i}(t), \mu_i, \alpha(t), \beta(t), \phi_i(t), \delta_i(t), \\ & \psi_i(t), \eta_i(t), \nu, \omega_i(t), \varpi_i(t)) \\ &= \frac{1}{T} \sum_{t=1}^T \sum_{i=1}^N q_i(t) \log_2(1 + \varphi_{d_i}(t) g_{d_i}(t)) \\ &+ \frac{1}{T} \sum_{t=1}^T \sum_{i=1}^N \mu_i \left[ q_i(t) \log_2(1 + \varphi_{d_i}(t) g_{d_i}(t)) \right. \\ &\left. - p_i(t) \log_2(1 + \varphi_{s_i}(t) g_{s_i}(t)) \right] \\ &+ \sum_{t=1}^T \alpha(t) \left[ 1 - \sum_{i=1}^N p_i(t) \right] + \sum_{i=1}^N \beta(t) \left[ 1 - \sum_{i=1}^N q_i(t) \right] \\ &+ \sum_{t=1}^T \sum_{i=1}^N \phi_i(t) [1 - p_i(t)] + \sum_{i=1}^T \sum_{i=1}^N \delta_i(t) p_i(t) \\ &+ \sum_{t=1}^T \sum_{i=1}^N \psi_i(t) [1 - q_i(t)] + \sum_{i=1}^T \sum_{i=1}^N \eta_i(t) q_i(t) \\ &+ \nu \left[ \bar{P}_{sum} - \frac{1}{T} \sum_{t=1}^T \sum_{i=1}^N (p_i(t) P_{s_i}(t) + q_i(t) P_{d_i}(t)) \right] \\ &+ \sum_{i=1}^N \omega_i(t) P_{s_i}(t) + \sum_{i=1}^N \varpi_i(t) P_{d_i}(t), \end{aligned} \quad (81)$$

where the Lagrange multipliers  $\mu_i$ ,  $\alpha(t)$ ,  $\beta(t)$ ,  $\phi_i(t)$ ,  $\delta_i(t)$ ,  $\psi_i(t)$ ,  $\eta_i(t)$ ,  $\nu$ ,  $\omega_i(t)$ ,  $\varpi_i(t)$  are chosen such that all the constraints are satisfied.

For simplicity, we omit time slot indexes here. Differentiate the Lagrangian function  $\mathcal{L}(\cdot)$  with regarding to  $p_i$  and  $q_i$ , and set them to zero. We have

$$-\frac{\mu_i}{T} \log_2(1 + P_{s_i} \frac{g_{s_i}}{\sigma_r^2 + \sigma_s^2}) + \alpha - \phi_i + \delta_i - \frac{\nu}{T} P_{s_i} = 0, \quad (82)$$

$$(1 + \mu_i) \frac{1}{T} \log_2(1 + P_{d_i} \frac{g_{d_i}}{\sigma_d^2}) + \beta - \psi_i + \eta_i - \frac{\nu}{T} P_{d_i} = 0, \quad (83)$$

respectively.

We first consider the selection scheme for the source nodes. If assuming the  $k$ -th source node  $S_k$  is selected, we have  $\delta_k = 0$  and  $\phi_i = 0$  for  $i \neq k$  according to the complementary slackness theorem [24]. Substituting them into (82), we get

$$T(-\alpha + \phi_k) = -\mu_k \log_2(1 + P_{s_k} \bar{g}_{s_k}) - \nu P_{s_k} \triangleq \Psi_k, \quad (84)$$

$$T(-\alpha - \delta_i) = -\mu_i \log_2(1 + P_{d_i} \bar{g}_{d_i}) - \nu P_{s_i} \triangleq \Psi_i, i \neq k. \quad (85)$$

where  $\bar{g}_{s_i} = \frac{g_{s_i}}{\sigma_r^2 + \sigma_s^2}$  and  $\bar{g}_{d_i} = \frac{g_{d_i}}{\sigma_d^2}$ . Since  $\phi_k \geq 0$ ,  $\delta_i \geq 0$ , we have  $\Psi_k \geq \Psi_i$ ,  $i \neq k$ , where  $\Psi_i$  denotes the selection

function for the source node  $S_i$ . Here, we can conclude that if  $p_k = 1$ ,  $\Psi_k \geq \max\{\Psi_i\}$ . Hitherto, we obtain the optimal scheduling scheme for the source nodes. In the similar way, if we set  $q_k = 1$ , we have

$$T(-\beta + \psi_k) = (1 + \mu_k) \log_2(1 + P_{s_k} \bar{g}_{s_k}) - \nu P_{s_k} \triangleq \Phi_k, \quad (86)$$

$$T(-\beta - \eta_i) = (1 + \mu_i) \log_2(1 + P_{d_i} \bar{g}_{d_i}) - \nu P_{d_i} \triangleq \Phi_i, i \neq k. \quad (87)$$

where  $\psi_k \geq 0$ ,  $\eta_i \geq 0$ , we have  $\Phi_k \geq \Phi_i$ ,  $i \neq k$ , where  $\Phi_i$  denotes the optimal decision function of the destination  $D_i$ . Hitherto, we have the optimal scheduling scheme for the destination nodes.

Furthermore, we need to determine the value of  $P_{s_i}$  and  $P_{d_i}$ . Differentiate the  $\mathcal{L}(\cdot)$  function with regard to  $P_{s_i}$  and  $P_{d_i}$ , and set them to zero. After differentiation we get

$$\frac{-\mu_i p_i}{T} \frac{g_{s_i}}{\ln(2)(1 + P_{s_i} \bar{g}_{s_i})} - \frac{\nu}{T} p_i + \omega_i = 0, \quad (88)$$

$$\frac{1 + \mu_i p_i}{T} \frac{g_{d_i}}{\ln(2)(1 + P_{d_i} \bar{g}_{d_i})} - \frac{\nu}{T} q_i + \varpi_i = 0. \quad (89)$$

Assume  $p_i \neq 0$  and  $q_i \neq 0$ , we get

$$P_{s_i} = \frac{-\mu_i}{\ln(2)(\nu - \omega_i T)} - \frac{1}{\bar{g}_{s_i}}, \quad (90)$$

$$P_{d_i} = \frac{1 + \mu_i}{\ln(2)(\nu - \varpi_i T)} - \frac{2}{\bar{g}_{d_i}}. \quad (91)$$

If there is a solution for (88), then (90) must be satisfied. In addition, we note that when the power  $P_{s_i}$  and  $P_{d_i}$  are nonzero, the Lagrangian multipliers  $\omega_i$  and  $\varpi_i$  will be zero and vice versa. Since the normalized power is always non-negative, we get

$$P_{s_i}(t) = \begin{cases} \frac{-\mu_i}{\ln(2)\nu} - \frac{1}{\bar{g}_{s_i}(t)}, & \text{if } \bar{g}_{s_i}(t) > \frac{\ln(2)\nu}{-\mu_i} \\ 0, & \text{otherwise} \end{cases} \quad (92)$$

Similarly, we get

$$P_{d_i}(t) = \begin{cases} \frac{1 + \mu_i}{\ln(2)\nu} - \frac{1}{\bar{g}_{d_i}(t)}, & \text{if } \bar{g}_{d_i}(t) > \frac{\ln(2)\nu}{1 + \mu_i} \\ 0, & \text{otherwise} \end{cases} \quad (93)$$

### APPENDIX D PROOF OF COROLLARY 3

The left hand side of (21) is the expectation of  $p_i(t) \log_2(1 + P_{s_i}(t) \bar{g}_{s_i}(t))$ , which is nonzero only when both  $p_i(t)$  and  $P_{s_i}(t)$  are nonzero. The conditions that  $p_i(t)$  and  $P_{s_i}(t)$  are nonzero can be obtained from (46) and (48) and are given by

$$\begin{cases} \bar{g}_{s_i}(t) > \frac{\ln(2)\nu}{-\mu_i}, \\ \xi_{s_j}^i > \bar{g}_{s_j}(t), \quad \forall j \neq i \end{cases} \quad (94)$$

where  $\xi_{s_j}^i$  is given by (56).  $p_i(t) \log_2(1 + P_{s_i}(t) \bar{g}_{s_i}(t))$  has to be integrated over the region given by (94) to obtain its expectation. This leads to the left side of (52). Similarly, the right hand side of (21) is the expectation of the  $q_i(t) \log_2(1 + P_{d_i}(t) \bar{g}_{d_i}(t))$ , which is nonzero only when both

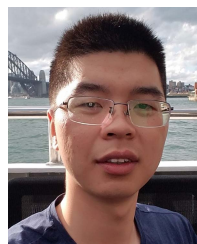
$q_i(t)$  and  $P_{d_i}(t)$  are nonzero. The conditions that  $q_i(t)$  and  $\gamma_{d_i}(t)$  are nonzero and can be obtained from (47) and (49). They are given by

$$\begin{cases} \bar{g}_{d_i}(t) > \frac{\ln(2)\nu}{1 + \mu_i}, \\ \xi_{d_j}^i > \bar{g}_{d_j}(t), \quad \forall j \neq i \end{cases} \quad (95)$$

where  $\xi_{d_j}^i$  is given by (57). To obtain the expectation,  $q_i(t) \log_2(1 + P_{d_i}(t)\bar{g}_{d_i}(t))$  has to be integrated over the region given by (95). This leads to the right side of (52). Following a similar procedure, we can obtain (53), which ends the proof.

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